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Solid Mechanics, Plasticity, and Limit Analysis

# DOUBLE PUNCH TEST FOR TENSILE STRENGTH OF CONCRETE

by  
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## ABSTRACT

This paper describes a new test technique for determining the tensile strength of concrete. In this test, a concrete cylinder (or cube) specimen is placed vertically between the loading platens of a testing machine and compressed by two steel punches located concentrically on the top and bottom surfaces of the cylinder (or cube). The relevant formula for computing the tensile strength in the new test is herein developed using the theory of perfect plasticity. It is shown that the necessary test arrangement, as well as the formula for computing the tensile strength of concrete, are very simple. The new test appears promising for practical use.

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## INTRODUCTION

The tensile strength of concrete is usually determined from indirect tensile tests (splitting tests on cylinders) rather than from direct pull tests on briquettes and bobbins or from flexural tests on beams (although flexural tests are valuable in connection with road and runway work). In countries where the compressive strength of concrete is determined from cubes rather than from cylinders, the tensile strengths have been obtained using a split cube or a cube specimen tested diagonally.

The drawbacks of direct pull tests include the difficulty in eliminating eccentricity of the line of action of the load and the development of stress concentrations near the gripping devices. Flexural tests are easier but the tensile strength determined in this way is significantly higher than the direct pull tensile strength. Indirect tensile tests are the most attractive because they enable similar specimens, and the same testing machine, to be used for both tensile and compressive strength tests. In addition, indirect tensile tests on cylinders give more consistent results with the measured strengths being between those of the other two tests. An appraisal of the splitting tests on cylinders has been given by Wright (1) and splitting tests on cube specimens have been described by Nilssons (2). Techniques used for the determination of tensile strengths have been discussed thoroughly by Malhotra (3). An

analytical study using a finite element method for various splitting tests has been reported recently by Davies and Bose (4).

A formula for computing the tensile strength of indirect tensile tests has been obtained from the theory of linear elasticity (5) and a plasticity treatment of this problem has been given recently by Chen (6). It is found that the result derived from the theory of perfect plasticity is identical to that derived from the theory of linear elasticity.

The success in applying the theory of perfect plasticity to the problem of the indirect tensile test suggests an alternative new testing technique for the determination of the tensile strength of concrete.

A double punch test is proposed here. In this test, a concrete cylinder is placed vertically between the loading platens of the test machine and compressed by two steel punches located concentrically on the top and bottom surfaces of the cylinder (Fig. 1). It is observed (6) that, although the specimen splits across the vertical diametric plane in a manner exactly similar to that observed in an indirect tensile test; the necessary test arrangement in obtaining the tensile strength of concrete may be reduced.

#### Calculation of Double Punch Tensile Strength

The work cited in Reference 6 is essentially an extension of the work described in Reference 7, in which the limit theorems

of the generalized theory of perfect plasticity (8) were applied to obtain the bearing capacity of concrete blocks or rock. The predicted bearing capacity of concrete blocks is found to be in good agreement with published test results. The approach is based on the assumption that sufficient local deformability of concrete in tension and in compression does exist to permit the application of the generalized theorems of limit analysis to concrete idealized as a perfectly plastic material (7). A Mohr-Coulomb failure surface in compression and a small but non-zero tension cut-off is utilized (see Fig. 2). In Fig. 2,  $f_c'$  and  $f_t'$  denote the simple compression and simple tension strength respectively, and  $c$  is cohesion  $\phi$  is the symbol of internal friction of the concrete.

The new approach has proved to be very fruitful for bearing capacity problems in concrete, for one thereby arrives at mathematical formulations which not only permit problems to be solved in a relatively simple mathematical form but also give promise of providing very satisfactory agreement with observations.

Since the behavior of a concrete block during a bearing capacity test is closely related to the behavior of a double punch test, the relevant formula of the double punch test can therefore be obtained directly from a simple modification of the results reported in Reference (7).

Fig. 3 shows diagrammatically an ideal failure mechanism for a double punch test on a cylinder specimen. It consists of

many simple tension cracks along the radial direction and two cone-shape rupture surfaces directly beneath the punches. The cone-shapes move toward each other as a rigid body and displace the surrounding material horizontally sideways. The relative velocity vector  $\delta w$  at each point along the cone surface is inclined at an angle  $\phi$  to the surface (7). The compatible velocity relation is also shown in Fig. 3. It is a simple matter to calculate the areas of the surfaces of discontinuity. The rate of dissipation of energy then is found by multiplying the area of each discontinuity surface by  $f_t'$  times the separation velocity  $2\Delta_r$  across the surface for a simple "tensile" crack or  $f_c' (1 - \sin \phi)/2$  times the relative velocity  $\delta w$  across the cone-shape rupture surface for a simple "shearing" (7). Equating the external rate of work to the total rate of internal dissipation yields the value of the upper bound on the applied load  $Q$

$$\frac{Q^u}{2a} = \frac{1 - \sin \phi}{\sin \alpha \cos (\alpha + \phi)} \frac{f_c'}{2} + \tan (\alpha + \phi) \left( \frac{bH}{2a} - \cot \alpha \right) f_t' \quad (1)$$

in which  $\alpha$  is the as yet unknown angle of the cone,  $a$  is the radius of the punch and  $b$  and  $H$  are the specimen dimensions (Fig. 3).

The upper bound has a minimum value when  $\alpha$  satisfies the condition  $\partial Q^u / \partial \alpha = 0$ , which is

$$\cot \alpha = \tan \phi + \sec \phi \left\{ 1 + \frac{\frac{bH}{2a} \cos \phi}{\frac{f_c'}{f_t'} \left[ \frac{1 - \sin \phi}{2} \right] - \sin \phi} \right\}^{\frac{1}{2}} \quad (2)$$

and Eq. 1 can be reduced to

$$\frac{Q^u}{\pi a^2} = f_t' \left[ \frac{bH}{a^2} \tan (2\alpha + \phi) - 1 \right] \quad (3)$$

Using typical values of  $f_c' = 10 f_t'$  and  $\phi = 30^\circ$ , and assuming  $2a = 1.5$  in.,  $2b = 6$  in. and  $H = 6$  in., the upper bound has a minimum value at the point  $\alpha = 11.2^\circ$ , and Eq. 3 gives

$$Q \leq Q^u = \pi (1.30 bH - a^2) f_t' \quad (4)$$

As concluded in Reference 7, the upper bound solution so obtained is in fact very close to the correct value. It seems therefore reasonable to take Eq. 5

$$f_t' = \frac{Q}{\pi (1.30 bH - a^2)} \quad (5)$$

as a working formula for computing the tensile strength in a double punch test.

It is important to note that earlier bearing capacity tests indicate that when the ratio  $b/a$  or  $H/2a$  is greater than 4 approximately [7], the local deformability of concrete in tension is not sufficient to permit the application of limit analysis. Rupture of the mortar or concrete penetrates progressively downwards from the tip of the cone-shape formed directly under the punch and crack propagation dominates. In such circumstances, the applied load  $Q$  becomes equal to



that of a double punch test with the ratio  $b/a = 4$  or  $H/2a = 4$ . Therefore, for any ratio  $b/a > 4$  or  $H/2a > 4$ , the limiting value  $b = 4a$  or  $H = 8a$  should be used in Eq. 5 for the computation of the tensile strength in a double punch test. For example, for the dimensions used in a cylinder compression test:  $2b = 6$  in. (15.30 cm) and  $H = 12$  in. (30.60 cm), assuming the same punch diameter  $2a = 1.5$  in. (3.80 cm), the appropriate value for  $H$  in Eq. 5 is 6 in. (15.30 cm) instead of the value 12 in. (30.60 cm).

Eq. 5 may be considered also valid for the case of a circular double punch on a square block specimen. However, the restrictions on the limiting value of the ratio  $b/a = 4$  (specimen width/punch diameter) or  $H/2a = 4$  should be taken into account in a similar manner.

The following example shows a typical double punch test for a cylinder specimen:  $Q = 26,500$  lb. (12 kg),  $2a = 1.5$  in. (3.80 cm),  $2b = 6$  in. (15.30 cm),  $H = 6$  in. (15.30 cm).

$$f'_t = \frac{26,500}{\pi [1.30 \times 3 \times 6 - (0.75)^2]} = 370 \text{ psi} \approx \frac{1}{12} f'_c \quad (6)$$

Table 1 shows the tensile strength computed from the results of a number of double punch tests reported recently by Chen and Hyland (9).

TABLE 1  
TENSILE STRENGTH COMPUTED FROM DOUBLE PUNCH TEST

SET	MAKE	CYLINDER HEIGHT H IN. (cm)	ULT. LOAD Q kip (kg)	$f'_t$ psi (kgf/cm <sup>2</sup> )	$\frac{f'_c}{f'_t}$
1	MORTAR	12 (30.60)	23.7 (10.7)	331 (23.2)	13.3
2		6 (15.30)	26.5 (12.0)	370 (26.0)	11.9
3		4 (10.20)	20.6 ( 9.4)	435 (30.6)	10.1
4	CON- CRETE	12 (30.60)	36.5 (16.6)	508 (35.7)	12.6
5		6 (15.30)	32.2 (14.6)	449 (31.6)	14.3
6		4 (10.20)	27.0 (12.3)	570 (40.0)	11.3

Punch diameter =  $2a = 1.5$  in. (3.80 cm)  
Cylinder width =  $2b = 6$  in. (15.30 cm)

### CONCLUSIONS

The tensile strength of concrete may be estimated by a double punch test proposed in this paper. The necessary test arrangement and the formula for computing the tensile strength of the new test are seen to be rather simple. It is stressed, however, that the magnitude of the exact difference between the double punch tensile strength and the "true" tensile strength of concrete has yet to be firmly established. The new test appears promising for practical use.

### ACKNOWLEDGEMENTS

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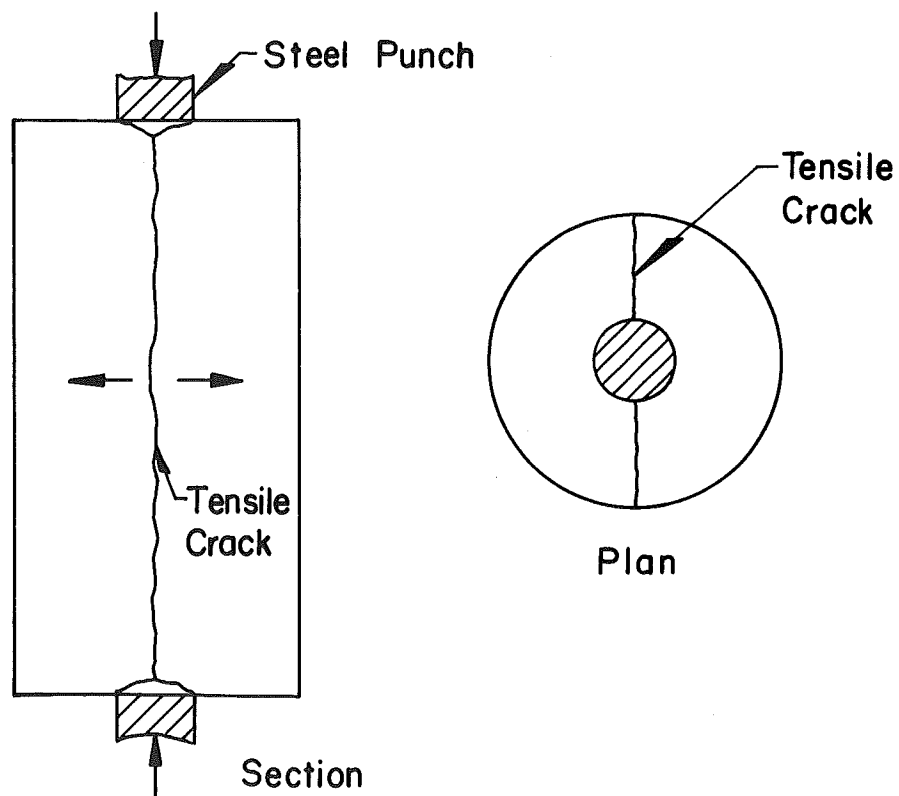
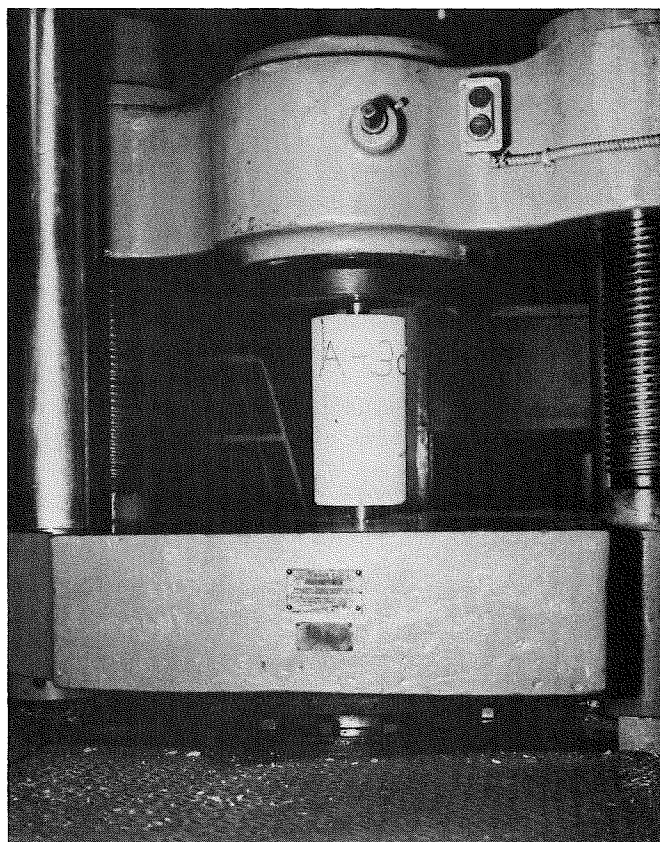


Fig. 1 A Double Punch Test

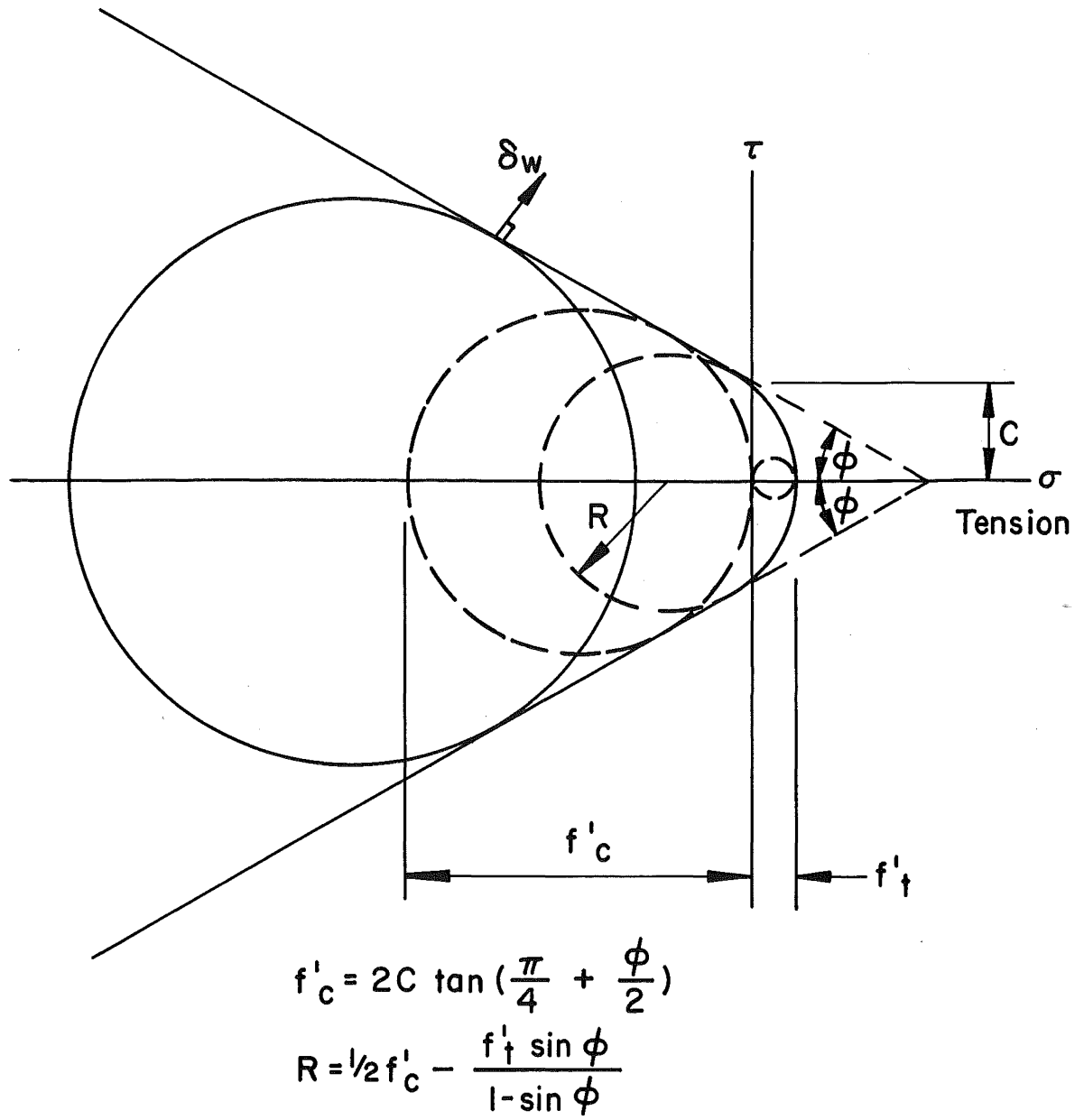


Fig. 2 Modified Mohr-Coulomb Criterion

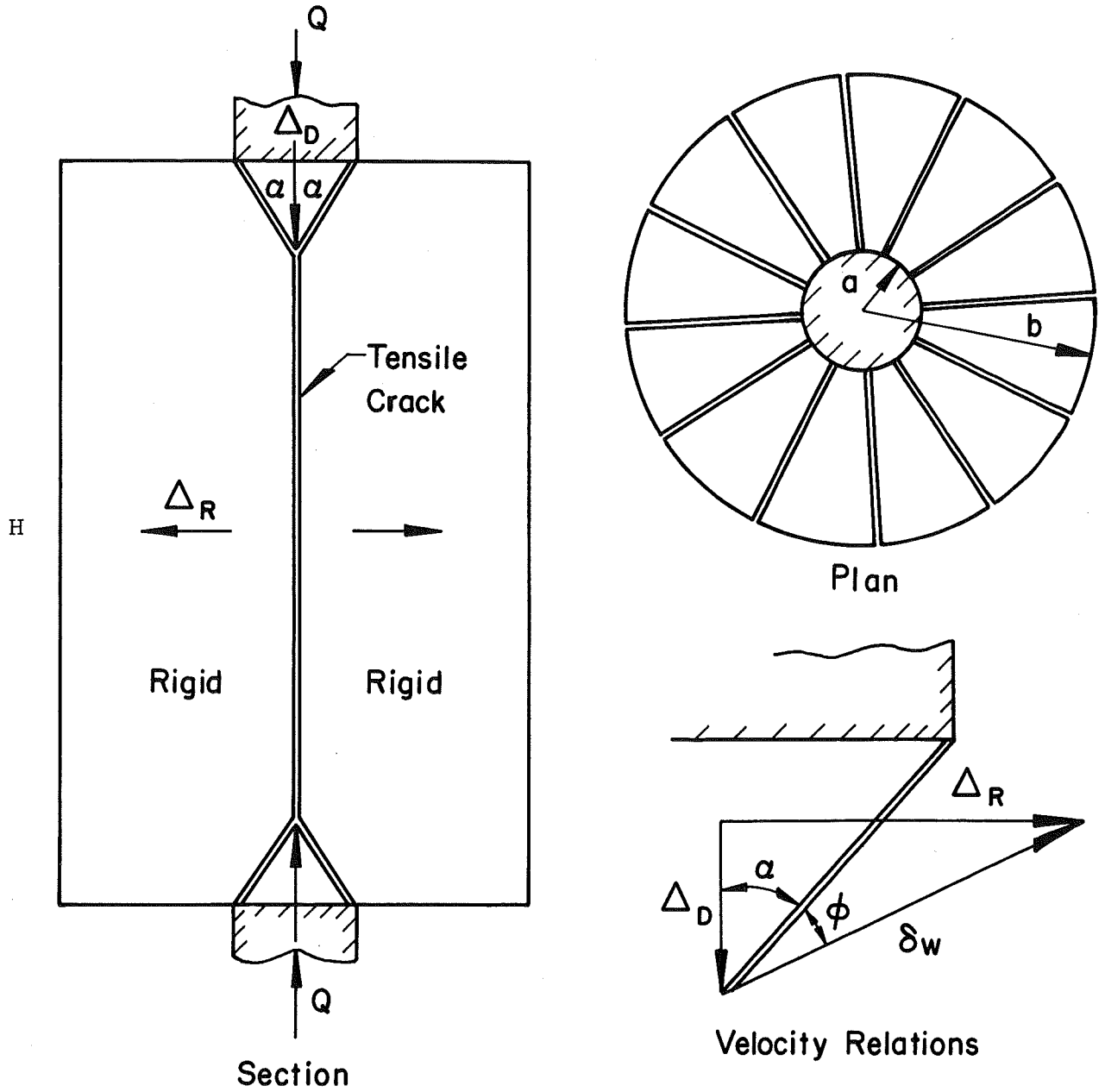


Fig. 3 Bearing Capacity of a Double Punch Test